Problem Set 3 — Due Friday, November 3, before class starts
For the Exercise Sessions on Oct 27

<table>
<thead>
<tr>
<th>Last name</th>
<th>First name</th>
<th>SCIPER Nr</th>
<th>Points</th>
</tr>
</thead>
</table>

Rules: You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Problem 1:

Find the maximum entropy density $f$, defined for $x \geq 0$, satisfying $E[X] = \alpha_1$, $E[\ln X] = \alpha_2$. That is, maximize $-\int f \ln f$ subject to $\int xf(x)dx = \alpha_1$, $\int (\ln x)f(x)dx = \alpha_2$, where the integral is over $0 \leq x < \infty$. What family of densities is this?

Problem 2:

What is the maximum entropy distribution $p(x,y)$ that has the following marginals?

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_{11}$</td>
<td>$p_{12}$</td>
<td>$p_{13}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_{21}$</td>
<td>$p_{22}$</td>
<td>$p_{23}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_{31}$</td>
<td>$p_{32}$</td>
<td>$p_{33}$</td>
</tr>
</tbody>
</table>

Problem 3:

(a) What is the parametric-form maximum entropy density $f(x)$ satisfying the two conditions

$$E[X^8] = a \quad E[X^{16}] = b$$

(b) What is the maximum entropy density satisfying the condition

$$E[X^8 + X^{16}] = a + b$$

(c) Which entropy is higher?
Problem 4:
Find the parametric form of the maximum entropy density $f$ satisfying the Laplace transform condition
\[ \int f(x)e^{-\alpha}dx = \alpha, \]
and give the constraints on the parameter.

Problem 5:
Let $Y = X_1 + X_2$. Find the maximum entropy of $Y$ under the constraint $E[X_1^2] = P_1$, $E[X_2^2] = P_2$:
(a) If $X_1$ and $X_2$ are independent.
(b) If $X_1$ and $X_2$ are allowed to be dependent.

Problem 6:
We learned in the course that as long as the set of feasible means is open then every such mean can be realized by an element of the exponential family. In the following verify this explicitly (by not referring to the above statement for the following scenario).
(i) Let $\phi(x) = (x^2)$.
(ii) Let $\phi(x)$ consist of all elements $x_i x_j$, where $i$ and $j$ go from 1 to $K$. 