1. a) \( \mathbb{E}(X) = p, \text{Var}(X) = p(1-p). \)
b) \( \mathbb{E}(X) = np, \text{Var}(X) = np(1-p). \)
c) \( \mathbb{E}(X) = \lambda, \text{Var}(X) = \lambda. \)
d) \( \mathbb{E}(X) = \frac{a+b}{2}, \text{Var}(X) = \frac{(b-a)^2}{12}. \)
e) \( \mathbb{E}(X) = \mu, \text{Var}(X) = \sigma^2. \)
f) \( \mathbb{E}(X) \) are \( \text{Var}(X) \) are not defined.

2. a) By integration by parts, one obtains:
\[
\mathbb{E}(X^4) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} x^3 \cdot x \exp\left( -\frac{x^2}{2\sigma^2} \right) \, dx = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} 3x^2 \cdot \sigma^2 \exp\left( -\frac{x^2}{2\sigma^2} \right) \, dx = 3\sigma^4
\]
b) \[
\mathbb{E}(\exp(X)) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} \exp\left( x - \frac{x^2}{2\sigma^2} \right) \, dx = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} \exp\left( -\frac{(x-\sigma)^2}{2\sigma^2} + \frac{\sigma^2}{2} \right) \, dx = \exp\left( \frac{\sigma^2}{2} \right).
\]
c) \[
\mathbb{E}(\exp(-X^2)) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} \exp\left( -x^2 \left( 1 + \frac{1}{2\sigma^2} \right) \right) \, dx = \frac{1}{\sqrt{2\pi\sigma^2}} \sqrt{1 + \frac{\pi}{2\sigma^2}} = \frac{1}{\sqrt{2\sigma^2 + 1}}.
\]

3. a) Using Cauchy-Schwarz’s inequality with \( X \) and \( Y = 1_{\{X>t\}}, \) we obtain
\[
\mathbb{E}(X 1_{\{X>t\}})^2 \leq \mathbb{E}(X^2) \mathbb{P}(\{X > t\}).
\]
On the other hand, we have \( \mathbb{E}(X 1_{\{X>t\}}) = \mathbb{E}(X) - \mathbb{E}(X 1_{\{X \leq t\}}) \geq \mathbb{E}(X) - t, \) therefore the result.
b) We check that
\[
\mathbb{P}(\{X > 0\}) = 1 - e^{-\lambda} \geq \frac{\lambda}{1 + \lambda} = \frac{\mathbb{E}(X)^2}{\mathbb{E}(X^2)}.
\]
(The central inequality follows from \( e^\lambda \geq 1 + \lambda, \forall \lambda > 0. \))

4. a) use \( \psi(x) = x^2 \) and \( \psi(x) = x^2 + \sigma^2 \) respectively.
b) \( \mathbb{P}(\{X \geq a\}) \leq \frac{\sigma^2 + b^2}{\sigma^2} = g(b). \) \( g \) has a minimum in \( b = \frac{\sigma^2}{\alpha} \) and at this point, \( g(b) = \frac{\sigma^2}{\alpha^2 + \sigma^2}. \)